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Ministry of Education and Training

NEW FOUNDATIONS FOR ONTARIO EDUCATION

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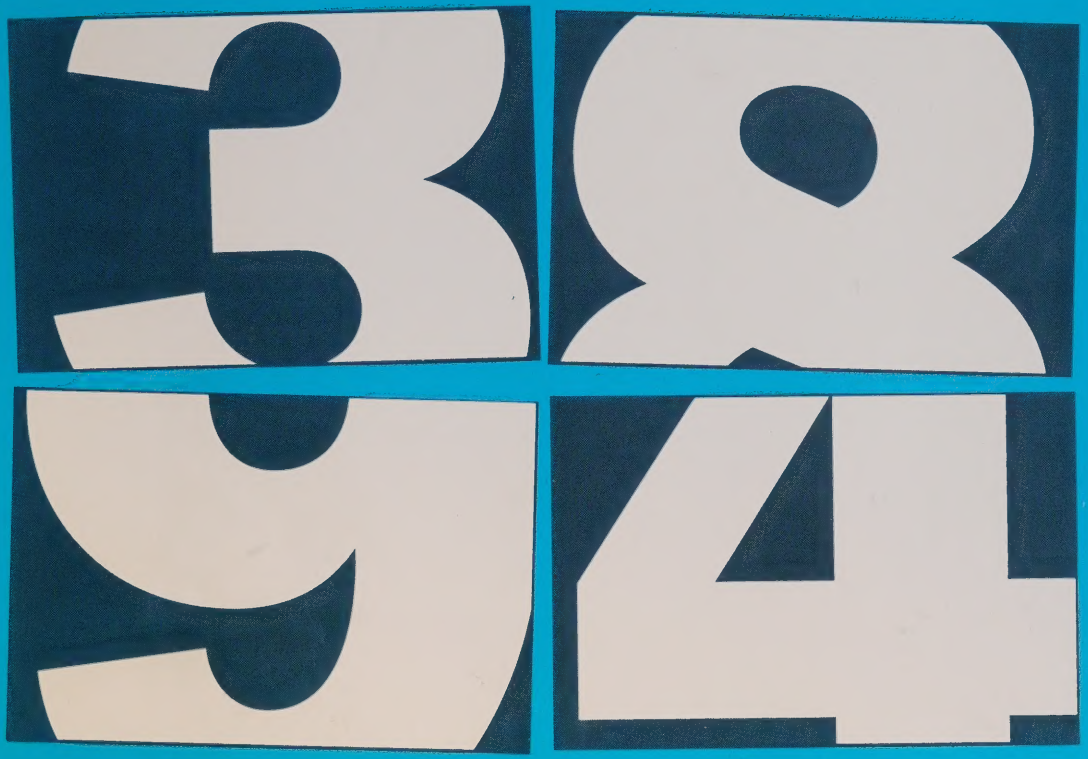
THE COMMON CURRICULUM



Provincial Standards
Mathematics, Grades 1-9

1995

FIELD-TEST VERSION

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Cette publication est également offerte en français sous le titre suivant : *Le programme d'études commun. Normes provinciales de mathématiques, de la 1re à la 9e année, 1995.*

Background

This document provides standards of performance in mathematics for the end of Grades 3, 6, and 9. The document is the result of close collaboration among mathematics teachers, Ministry of Education and Training officials, and representatives of mathematics associations in Ontario; it also incorporates materials from school boards in Ontario and from other jurisdictions and countries. The Ministry of Education and Training is pleased to acknowledge the co-operation and assistance of the teachers, researchers, and co-ordinators who have contributed to the development of these standards.

Figure 1: The Use of Outcomes and Standards in Assessing, Reporting, and Improving Student Achievement

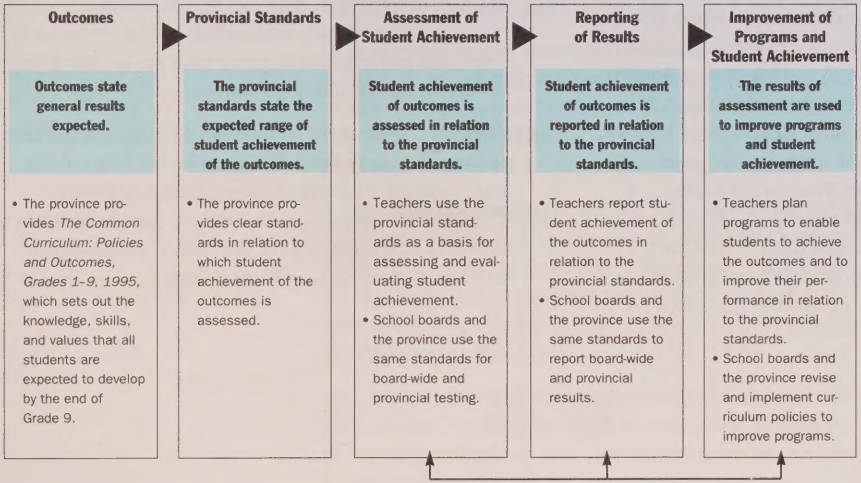

Outcomes
Outcomes state general results expected.
• The province provides *The Common Curriculum: Policies and Outcomes, Grades 1–9, 1995,* which sets out the knowledge, skills, and values that all students are expected to develop by the end of Grade 9.
Provincial Standards
The provincial standards state the expected range of student achievement of the outcomes.
• The province provides clear standards in relation to which student achievement of the outcomes is assessed.
Assessment of Student Achievement
Student achievement of outcomes is assessed in relation to the provincial standards.
• Teachers use the provincial standards as a basis for assessing and evaluating student achievement.
• School boards and the province use the same standards for board-wide and provincial testing.
Reporting of Results
Student achievement of outcomes is reported in relation to the provincial standards.
• Teachers report student achievement of the outcomes in relation to the provincial standards.
• School boards and the province use the same standards to report board-wide and provincial results.
Improvement of Programs and Student Achievement
The results of assessment are used to improve programs and student achievement.
• Teachers plan programs to enable students to achieve the outcomes and to improve their performance in relation to the provincial standards.
• School boards and the province revise and implement curriculum policies to improve programs.

Figure 1 shows how the outcomes and standards are used in assessing, reporting, and improving student achievement.

A Framework for Assessment

Assessment is the process of gathering evidence about a student's knowledge, skills, and values and of making inferences based on that evidence for a variety of purposes. These purposes include: making instructional decisions; monitoring student progress; evaluating student achievement in terms of defined criteria; and evaluating programs.

Assessment takes place within a particular context or framework of expectations. In this document, the Ministry of Education and Training seeks to provide such a framework using the organization and format described below.

The document has four main parts: Part I, "Assessment Strategies", discusses assessment methods that are recommended to supplement the traditional paper-and-pencil testing. Part II, "Key Components of Mathematics Learning", discusses communication, reasoning, problem solving, connections, and technology as integral components of mathematics learning.

For the purposes of the discussion in Parts III and IV, the topics taught in the mathematics program are grouped into six specific mathematical areas, or "strands".

These are:
- problem solving and inquiry;
- number sense and numeration;
- geometry and spatial sense;
- measurement;

– patterning and algebra;
– data management and probability.

Part III, "Strands of Mathematics", outlines important characteristics of each strand and some appropriate methodologies for teaching it.

Part IV, "Assessment Standards", outlines in detail a framework that can be used to measure students' performance in each strand of the program at the end of Grades 3, 6, and 9.

The components of this framework are as follows:

1) ***Outcomes from*** **Policies and Outcomes, Grades 1–9.** For each strand, the expected outcomes from *The Common Curriculum: Policies and Outcomes, Grades 1–9* are stated. These identify the mathematical understanding students should have acquired and the operations they should be able to perform. Collaborative effort is required by teachers of all grades to ensure that students achieve these outcomes.

   In addition, the more specific outcomes on which the mathematics standards are based are described for each strand at the end of Grades 3, 6, and 9 in the paragraph that begins with "More specifically, students will:". It is the responsibility of all teachers in Grades 1 to 9 to work together to deliver an effective mathematics program that is appropriate for all students. Teachers of Grades 1, 2, and 3 are encouraged to work together to develop a program based on the outcomes described in the Grade 3 section. Similarly, teachers in Grades 4 to 6 and Grades 7 to 9 would also work together to develop programs for those grades.

2) ***Characteristics of Student Performance.*** For each strand, descriptions of student performance are provided at four levels. These are general descriptions of the degrees of proficiency students exhibit. They may be helpful in reporting procedures, where a description of the level is required.

3) ***Sample Performance Indicators.*** A list of "Sample Performance Indicators" is given for each level of performance. These offer specific examples of what students know and can do at each of the four levels. While efforts have been made to provide substantial lists of performance indicators, the lists are not intended to be exhaustive or restrictive.

   It should be noted that problem solving, which is a central focus of mathematics learning, is treated both as a strand of the mathematics program and as an integral part of all other strands. The Sample Performance Indicators described for problem solving are applicable to all grades and strands. For this reason, as well as to avoid unnecessary repetition, aspects such as risk-taking, attitudes to work, and participation in group work are included only in this strand and not in the others.

4) ***Sample Assessment Activities.*** Examples of the alternative assessment strategies described in Part I of this document are illustrated for each strand and level. Teachers are encouraged to use these sample activities to familiarize themselves with approaches to assessment and instruction that may be new to them.

Descriptions of performance at four levels are provided for each strand at the end of Grade 3, Grade 6, and Grade 9, as shown in Figure 2 (see page 6). Each description includes:

- a level from 1 to 4 which indicates the degree of proficiency demonstrated;
- a general description of what students at that level can do in the mathematical area;
- a list of Sample Performance Indicators providing specific examples of what students at that level can do in the area.

What the Standards Mean

Students do not learn things in the same order or at the same rate. In addition, the nature and context of performance will be very different for an eight-, eleven-, or fourteen-year-old. The four levels demonstrate the range of performance for each of the three identified grades. It is important to recognize that the levels do not represent a simple continuum – for example, Level 3 performance in Grade 3 is not equivalent to Level 1 performance in Grade 6.

Of the four categories described, Level 2 and Level 3 are considered to be within the expected range of standards for student performance in Ontario. Performance described in Level 1 is below the expected standard; performance described in Level 4 is beyond the expected standard.

It is unlikely that the performance of any one student will fall exclusively into a single category; rather, a student's level of performance will vary according to the strand of mathematics. It is important to recognize that the Standards of Performance represent descriptions of student performance, not descriptions of students.

The Standards of Performance describe high but reasonable expectations. An effort has been made to blend observations of current performance with desired performances, in order to set standards that are future-oriented but realistic.

How the Standards Can Be Used

The Provincial Standards for Mathematics are broad standards for the province designed to help in assessing students' learning. They set worthwhile goals for all, by describing expected provincial levels of performance. Their primary aim is to make a clear statement to teachers, parents, and students about expected results.

Teachers may use the more specific outcomes as a basis for team planning in divisions. They may use statements of the Standards of Performance, together with the Sample Performance Indicators, to assess an individual student's work and to plan with the student and parents for improvement. As well, teachers may use the standards as a reference in reporting student performance to parents.

Parents may use the Provincial Standards of Performance to assess their children's performance. School boards and the Ministry of Education and Training may use the Standards of Performance as a basis for program evaluation.

Figure 2: Framework for Mathematics Standards

	Strands						
Level	**Problem Solving and Inquiry**	**Number Sense and Numeration**	**Geometry and Spatial Sense**	**Measurement**	**Patterning and Algebra**	**Data Management and Probability**	
4							Students performing beyond the expected range of standards set for the grade.
3							**Standards of Performance** Students performing within the expected range of standards set for the grade.
2							
1							Students not yet performing within the expected range of standards set for the grade.

Outcomes-based education stresses process. Outcomes are the observable/measurable knowledge, skills, and values that students are expected to have acquired and developed at certain key stages of their schooling. They describe what students should know, should be able to do, and should value as a result of their learning experiences. Content becomes the tool for attaining outcomes. This shift towards outcomes-based education requires a modification of some existing assessment practices.

Increased attention should be given to a broad range of assessment practices such as:
- assessing what students know and how they think about mathematics;
- focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
- assessing student performance in a variety of ways, including written, oral, and demonstration forms;
- using calculators, computers, and manipulatives;
- recognizing such attitudinal outcomes as motivation and appreciation;
- assessing the process as well as the product.

Tests are one way of determining what students have learned. But mathematical competence involves such characteristics as communicative ability, problem-solving ability, higher-order thinking ability, creativity, persistence, and curiosity. A variety of assessment strategies should be used to provide students with opportunities to demonstrate these characteristics.

The following assessment strategies are recommended in order to supplement the traditional pencil-and-paper testing: performance assessment, open-ended questions, investigations, journals, observations, conferences and interviews, portfolios, and student self-assessment.

Performance Assessment

Performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing them, interviewing them, and examining what they produce to assess what they actually know and can do. Performance assessment focuses on the process as well as the product. It provides contexts that have relevance to the students' lives and enhances motivation as students work on real tasks.

Performance assessment gives information about a student's ability to:
- use mathematical concepts and skills;
- reason soundly and raise questions;
- think flexibly, changing strategies when a particular approach does not work;
- use manipulative materials and equipment, calculators, and computers;
- work in a group;
- persist, concentrate, and work independently;
- communicate and use mathematical language through discussing, writing, and explaining his or her ideas and questions;
- observe carefully to infer and formulate hypotheses;
- design and conduct experiments and investigations.

Open-Ended Questions

"Open-ended" means having no predetermined limit or boundary. Students are asked to respond to questions or solve problems to which a variety of successful responses are possible.

Open-ended questions give information about a student's ability to:
- recognize the essential points of the problem;
- organize and interpret information;
- make generalizations;
- write for a given audience;
- understand basic concepts;
- use appropriate mathematical language and representation;
- demonstrate originality and/or creativity.

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving.

Investigations give information about a student's ability to:
- identify and define a problem;
- make a plan;
- create and interpret strategies;
- collect and record needed information;
- organize information and look for patterns;
- persist, looking for more information if needed;
- discuss, review, revise, and explain results.

Journals

A mathematics journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions.

Journals allow opportunities for students to:
- formulate, organize, internalize, and evaluate concepts;
- clarify thinking;
- identify their own strengths, weaknesses, and interests;
- reflect on new learning;
- use the language of mathematics to describe learning.

The information provided by journals can be useful to teachers in modifying the program to meet individual needs.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups.

Systematic, ongoing observation gives information about students':
- attitudes towards mathematics;
- feelings about themselves as learners of mathematics;
- specific areas of strength and weakness;
- preferred learning styles;
- areas of interest;
- work habits;
- social development;
- development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide.

Teachers should develop a realistic plan for observing students. The plan might incorporate opportunities to:
- observe a small number of students each day;
- focus on one or two aspects of development at a time;
- tape-record or videotape student activities for later analysis.

Conferences and Interviews

An interview includes a planned sequence of questions, whereas a conference implies discussion, with student and teacher sharing ideas. The main purpose of both conferences and interviews is to explore the student's mathematical thinking and to assess the student's level of understanding of a particular concept or procedure.

Conferences and interviews give information about students' ability to:
- explain the process used to arrive at a solution;
- justify their thinking (i.e., explain and support their reasoning);
- suggest an alternative strategy;
- discuss their likes and dislikes;
- discuss their perceived strengths and weaknesses.

Techniques that contribute to a successful conference/interview include:
- using prepared questions;
- putting students at ease;
- explaining that you are interested in the process of reasoning followed by the student;
- posing a problem;
- taking notes;
- being a good listener;
- being non-judgemental;
- doing instructional intervention in a separate setting.

Most conferences/interviews should be brief and informal and should occur naturally in the course of everyday mathematical experiences.

Portfolios

A portfolio is a showcase for student work, a place where many types of work can be collected. The portfolio provides a comprehensive view of the student's progress in, attitudes towards, and understanding of mathematics.

A portfolio may include the following examples of student work:
- written descriptions of investigations;
- descriptions and/or analyses of problem situations;
- responses to open-ended questions;
- homework problems;
- photographs of a project;
- group reports;
- video, audio, or computer-generated examples of work;
- personally chosen work, with reasons for the choice;
- excerpts from the student's mathematics journal;
- a description of how the student feels about mathematics;
- a student self-report on what has been learned.

Student portfolios can provide:
- evidence of performance;
- evidence of growth or change over a period of time;
- a permanent record of student performance;
- opportunities for students to practise assessing and selecting their own work.

Student Self-Assessment

Student self-assessment promotes the development of metacognitive ability (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and to become independent thinkers. Self-assessment can be done by means of a questionnaire, following a co-operative activity or project, asking how well the group functioned, or through daily writing in a journal.

Teachers can use student self-assessments to determine:
- whether there is change and growth in the student's attitudes, mathematical understanding, and achievement;
- whether a student's beliefs about his or her performance correspond to actual performance;
- whether the student and the teacher have similar views of expectations and criteria for evaluation.

Assessing the Performance of Exceptional Students

Exceptional students include gifted students, students with special communication needs, and/or students requiring physical, social, emotional, or intellectual support. Exceptional students identified by an Identification, Placement, and Review Committee (IPRC) are provided with program modification, resources, and specialized personnel to enable them to achieve the learning outcomes outlined in *The Common Curriculum: Policies and Outcomes, Grades 1–9* and realize their full potential.

Like other students, exceptional students may perform at different levels for different types of mathematics activities. For example, some exceptional students may achieve at a higher level in geometry and spatial sense than in number sense and numeration. By building on student strengths and by using an integrated approach to mathematics, teachers can help exceptional students to perform to the best of their abilities.

The standards have been designed to take into consideration the needs of exceptional students and should be used as a guide during program development, assessment, and modification.

Sample Problem-Solving Activity (Grade 3)

You have this amount of change:
11 pennies, 4 nickels,
12 dimes, 6 quarters.

These items are for sale :

Bouncy ball	$0.25
Candy	$1.20
Small poster	$0.90
Pen	$2.15

Use this information to make up a problem.

Problem solving, communication, reasoning, connections, and technology are central and essential components of mathematics learning. The following descriptions of these components apply throughout the Formative and the Transition Years.

The introductory discussions of each component are adapted from the document *Focus on Renewal of Mathematics Education* (1993), prepared by the Ontario Association for Mathematics Education (OAME) and the Ontario Mathematics Coordinators Association (OMCA).

Problem Solving

Students use their mathematics knowledge and skills to help them find solutions to problems in their daily lives and to make decisions based on those solutions. Problem solving should be the central focus of the mathematics curriculum. It is the process by which students experience the power and usefulness of mathematics in the world around them. It is a method of inquiry and application that should be interwoven with the other five strands of mathematics to give students a consistent context for learning and applying mathematics.

Classroom instruction should provide students with wide-ranging and varied problem-solving experiences in a climate that encourages and supports students' efforts. Ideally, students should share their thinking and strategies with other students and with teachers. They should learn a variety of ways of representing and solving problems. In addition, they should value the process of solving problems as much as they value the solutions. Students should have many experiences that involve posing problems based on real-world activities, on organized data, and on equations.

Although problems will usually be based on concrete and empirical situations, the program should strike a balance between problems that apply mathematics to the real world and problems that arise solely from the investigation of mathematical ideas. The mathematics curriculum should also engage students in some problems that take an extended effort to solve. These problems may be part of group projects that require students to use available technology and to undertake co-operative problem solving and discussion.

Computers and calculators are powerful problem-solving tools. The power to compute rapidly and to investigate relationships among variables, using spreadsheets and graphing programs, can help students to meet and even seek out mathematical challenges with increasing resourcefulness and independence.

Communication

Mathematics is a language that must be made meaningful to students if they are to communicate mathematical ideas and apply mathematics effectively. Students' general communicative ability helps them to understand the language of mathematics. Their everyday language is a bridge that enables them to translate their informal, intuitive notions into the abstract language and symbolism of mathematics. Communication also plays a key role in helping students to make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas and, as a corollary, to gain a sense of the value of mathematics both as a type of communication system and as a tool. When students understand that one representation can describe many situations, and that some ways of representing a problem are more helpful than others, they begin to understand the power, flexibility, and usefulness of mathematics.

It is important for students to have opportunities to "talk mathematics". Interacting with classmates helps students to construct knowledge, learn to think about ideas in a variety of ways, and clarify their own thinking. Writing about mathematics – for example, describing how a problem was solved – also helps students to clarify their thinking and develop deeper understanding.

Sample Communication Activity (Grade 6)

Students should be encouraged to write about their mathematics experiences.

Dear Juanita,

Thanks for explaining your group's math project on networks to us. It gave me an idea on how to set up a topological map model for our group. I remember ...

Students need to be actively involved in "doing mathematics". The activities of exploring, investigating, describing, and explaining mathematical ideas promote the development of communication skills. Teachers facilitate this development when they pose probing questions and invite students to explain their thinking. The idea that students should learn mathematics through activities that have relevance to their own lives is implicit in this discussion. That is, students need to see mathematics as an effective means for understanding, describing, and responding to the world around them.

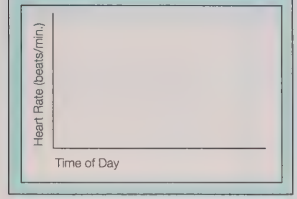
As students' ability to reason abstractly continues to develop, so too should their ability to communicate mathematically. The more effectively students can communicate their knowledge and insight, the more readily teachers can assess their learning through listening and observation.

Reasoning

Reasoning is fundamental to understanding and doing mathematics. Our goal is to help students develop confidence in their ability to do mathematics and see that they have control over their own success or failure. Students' sense of autonomy develops as they become aware of their ability to reason logically and to justify their thinking. Conjecturing and demonstrating the validity of conjectures are central to the creative process of doing mathematics. An emphasis on reasoning must pervade all classroom activity.

Sample Reasoning Activity (Grade 9)

Sketch a graph of your heart rate over the course of a day. Explain your reasoning.



Students need experience with a variety of types of problems and a reasonable amount of time to learn how to construct convincing arguments during problem-solving activities and how to evaluate the arguments of others. A climate should be established in the classroom that places critical thinking at the heart of instruction. All members of the class should be encouraged to question, react to, and elaborate on both the teacher's and the students' statements. For such a positive spirit of inquiry to prevail, all members of the class need to express genuine respect and support for one another's ideas.

The development of the ability to reason logically is tied to the intellectual and verbal development of students. Students begin as concrete thinkers who depend on a physical or concrete context to support their reasoning. Increasingly through the Formative and the Transition Years, students will become more capable of formal reasoning and abstraction. Even the most advanced students, however, may continue to benefit from the use of concrete materials to support their reasoning; this is especially true for spatial reasoning.

To develop their mathematical reasoning ability, students must have opportunities to explore, speculate, validate, and convince others. A classroom that provides challenging mathematical experiences and appropriate materials and tools will best meet this goal.

Connections

Students need to see how mathematical ideas are related to one another, to other subjects, and to everyday life. The mathematics curriculum should not be viewed as made up of discrete strands – such as number, or space – that can be taught in isolation from one another. It is important for students to connect ideas both within and among the areas of mathematics. Students need to view mathematics as an integrated whole and to recognize its usefulness and relevance both inside and outside school.

In classrooms where connections are emphasized, linking ideas, themes, or processes are used to ensure continuity between lessons. Lessons frequently extend over several days so that connections can be explored and discussed, and generalizations formulated. Once introduced, a topic is referred to regularly and related to other aspects of the mathematics program. Students are asked to compare and contrast the different concepts and procedures. They are helped to develop abstract ideas out of concrete experiences, and they learn different ways of representing a problem or concept.

Students also need to understand how they can use mathematics to learn in other subject areas and to function in daily life. Teachers should collaborate with staff members who are expert in other subject areas so that cross-curricular learning can occur. Students should be given opportunities to explore practical applications of mathematics. Such explorations will help them understand that mathematics is a powerful and flexible tool they can use to solve a wide variety of problems, to describe and model real-world phenomena, and to communicate complex ideas and information.

Technology

The teacher of today must increasingly value and encourage the use of a variety of tools for the learning of mathematics. Students at all grade levels must have access to appropriate manipulative materials so that each student can develop an understanding of new concepts. Students must also learn which tools are appropriate for particular tasks. The decision to use a particular tool may be an important part of the solution to a problem.

Teachers and students should have access to appropriate technology. In particular, every classroom should have at least one computer that is available for student use. Additional computers should be available for small-group, and whole-class use. Teachers and students should use computers as tools to assist with the exploration and discovery of concepts, with the process of deriving abstract mathematical ideas from concrete experiences, with the practice of skills, and with the process of problem solving. To be appropriate, software should enable students to develop and deepen their understanding of new concepts, rather than simply drilling them on facts or skills. Software such as databases, spreadsheets, and geometric drawing programs should be available to all classrooms.

Each student should have a calculator with functions that will help him or her perform the tasks required by the curriculum. Teachers should model the appropriate use of calculators as a problem-solving tool.

For purposes of organization of the Standards of Performance, the following strands of mathematics have been used:

- problem solving and inquiry;
- number sense and numeration;
- geometry and spatial sense;
- measurement;
- patterning and algebra;
- data management and probability.

With the exception of the strand "problem solving and inquiry", Standards of Performance for the end of Grade 3, Grade 6, and Grade 9 are provided for each strand. For "problem solving and inquiry", a single set of standards is provided that applies across the grades and strands.

The following section presents a discussion of the nature of each strand, including relevant methodological and developmental considerations. The discussion is adapted from the document *Focus on Renewal of Mathematics Education.*

Problem Solving and Inquiry

Inquiry Learning

"Children arrive in school with open, inquiring minds. They are already familiar with the inquiry process through their experiences with play, and their most important need in the Primary years is to have opportunities to continue their natural inclination towards inquiry learning." [1]

Inquiry learning is natural to young children. Primary teachers are familiar with and use the seven main stages of the inquiry process (exploring, inquiring, predicting possibilities, planning and collecting, deciding, communicating, evaluating) to help students to become responsible for their own learning.

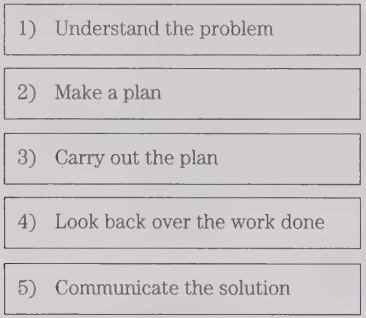
A Problem-Solving Model

Another form of the inquiry model is a multi-step process based on George Polya's research (understand the problem; make a plan; carry out the plan; look back; communicate the solution).[2] This model provides problem solvers with a framework within which to tackle problems. It is recommended that this general model for mathematics not be directly introduced to most children before Grade 3. The model, however, can be useful as a further mental framework for Primary teachers as they plan the classroom environment, take advantage of incidental opportunities for learning, and pose questions to help students become more successful problem solvers. As students progress through the grades, direct instruction using this model should occur. A poster of the model should be prominent in the classroom for reference during instruction and problem-solving activities.

1. Ministry of Education, Ontario, *Shared Discovery: Teaching and Learning in the Primary Years* (Toronto: Ministry of Education, Ontario, 1985), p. 9.

2. George Polya, *How to Solve It: A New Aspect of Mathematical Method*, 2nd ed. (1945; reprint, Princeton, N.J.: Princeton University Press, 1973).

Figure 3: Problem-Solving Model



Problem Solving as a Focus

Students should solve problems on a regular and frequent basis. Problem solving should not be viewed or presented as an isolated unit of study; rather, it should provide the context in which mathematics skills and concepts are learned. In a classroom where problem solving is an integral part of instruction, there is an emphasis on having students explore a variety of different kinds of problems (with a special focus on problems where the solution is not immediately evident) and on encouraging students to share their thinking. In a supportive classroom setting, solving problems in different ways is applauded, the process is valued as much as the solution, and students are actively involved in posing problems based on real-world activities.

Problem Solving as a Process

Students should have opportunities to work both individually and co-operatively on problems that are open-ended, that involve assumptions and limitations, and for which multiple solutions may be possible. Typical arithmetic problems are restricted – there is usually one correct answer and one way to reach that answer. A program involving only arithmetical problem solving would present students with a very limited view of the problem-solving process.

Problem solving is a process that may involve starting and stopping, failure and success, consideration and rejection, testing and exploration. It is important for teachers to model the process of problem solving in the classroom. Students sometimes have the impression that successful problem solvers just know what to do and can come up with a solution right away. If the teacher models or thinks out loud while solving a problem, and encourages students to do the same, all students may acquire a better appreciation of the process.

Number Sense and Numeration

Number Sense

Number sense has been described as

> an intuition about numbers that is drawn from all the varied meanings of number. It has five components:
> 1. Developing number meanings ....
> 2. Exploring number relationships with manipulatives ....
> 3. Understanding the relative magnitudes of numbers ....
> 4. Developing intuitions about the relative effect of operating on numbers ....
> 5. Developing referents for measures of common objects and situations in their environment ...[3]

All students require purposeful classroom experiences that will assist them in developing number sense. Appropriate experiences would include opportunities for students to

3. Working Groups of the Commission on Standards for School Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1989), pp. 39-40.

manipulate physical objects and use their own language to explain their thinking.

Students with good "operation sense" recognize situations in the real world where a particular arithmetical operation would be useful. They can also construct appropriate models and describe the steps of an operation, relationships among operations, and the effects of an operation on a pair of numbers.

Mental Mathematics and Computational Estimation

Students should develop mastery of basic one-digit computational facts by the end of Grade 6. To meet this goal, students require time to identify relationships between numbers and to develop efficient thinking strategies. Mastery is a critical component of fluency with paper-and-pencil and mental computation, as well as estimation.

Techniques of mental mathematics should be introduced gradually along with concepts of place value and paper-and-pencil computations. For example, the mental skill of adding numbers ending in zero, such as 20 + 30 + 60, can be learned by a student who understands the place value and can add 2 + 3 + 6.

After students have established a firm base in mental mathematics, the systematic teaching of strategies of computational estimation should occur. Computational estimation should include not only applications involving whole numbers and decimals, but also those involving fractions and per cent. In fact, early experiences in estimating with per cent and fractions help to establish related number sense.

It is essential for students to develop strong skills and strategies in mental mathematics and computational estimation, in order to judge the reasonableness of answers produced by a calculator or computer.

Paper-and-Pencil Computations

Many paper-and-pencil computations, such as long division, operations with large numbers, or calculation of the square root of a number, have become less used in light of the availability of calculators and computers. Emphasis on estimation supported by competence in mental calculation will enhance a student's ability to cope in our technological society.

Students (and adults) may continue to need some facility with paper-and-pencil computations, however. The following guidelines are recommended as being reasonable maximum limits for this kind of computation by the end of Grade 6:

- addition: three four-digit numbers
- subtraction: two four-digit numbers
- multiplication: a two-digit number by a two-digit number
- division: a three-digit number by a one-digit number (including short division)

It is important for paper-and-pencil computational procedures to be introduced through the use of concrete materials, and for students to develop skill through practice in context. It is inappropriate to engage students in pages of isolated drill of computational skills.

In the Transition Years, students who have achieved the maximum required levels should be given opportunities to maintain them through practice in context; students who have not achieved the required levels should be given opportunities to improve their skills.

Fractions and Rationals

Concepts and operations with fractions should be introduced using concrete materials. Models such as physical materials and diagrams should be used to relate fractions to decimals, to find equivalent fractions, and to explore operations with

fractions and decimals. Fraction symbols should be introduced only when students have developed the concepts and oral language to make the symbols meaningful.

In the Formative Years, the focus should be on developing fraction sense, a process that includes such things as: representing fractions concretely, pictorially, and symbolically; identifying, generating, and applying equivalent fractions; estimating with fractions and mixed numbers; and ordering fractions and mixed numbers. Operations with fractions should be limited to addition and subtraction, and to multiplication involving a fraction of a whole number. Formal procedures should be introduced only after students have had significant opportunities to develop understandings using concrete materials. In operations with whole numbers, there is a recognized multi-stage process that leads students from the concrete to the abstract, or from base-10 materials to the formal algorithms. Similarly, as students explore fractions with concrete materials such as pattern blocks, the notion of addition and subtraction becomes apparent. For example, where a yellow hexagon equals one whole, a green triangle has a value of $\frac{1}{6}$. Simple counting of two triangles represents $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$. If four green triangles are counted out to represent $\frac{4}{6}$, and one is removed, the operation of $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$ has been performed. It is important for students to have concrete experiences such as these before they attempt formal operations with fraction symbols.

It is recommended that, initially, simple denominators such as 2, 3, 4, 5, and 10 be used. As students gain more experience and skill in working with fractions, denominators such as 6, 8, and 12 can be included.

In the Transition Years, the focus should be on using fractions in the context of ratios, rates, and per cent. As well, students should extend their fraction sense to include skill in operations with fractions. (It should be remembered, however, that the most frequent real-life use of fractions involves estimation.) Signed fractions (rational numbers) can be introduced in Grade 9, but only simple calculations should be expected.

Use of the Calculator

In the Formative Years, calculators should be used for investigations, problem-solving activities, and real-life applications in which the complexity of the numbers is greater than the computational skills of the students. In the Transition Years, calculators should be used except when the purpose of an activity is to develop paper-and-pencil computational skills.

When using calculators in all grades, the focus should be on the development of the ability to use the calculator appropriately and effectively and to judge the reasonableness of an answer produced. Students must learn to decide whether a calculator is needed for a particular computation, or whether mental mathematics is more appropriate. The calculator is a powerful aid to computation, but its use must not be allowed to interfere with the intelligent application of good number sense and numeration skills.

Decision Making in Computation

It is essential for all students to have access to a wide range of computational tools, including concrete materials, mental mathematics, estimation, paper-and-pencil procedures, and calculators. During the problem-solving process, students will decide which tool is most appropriate in a given set of circumstances.

Geometry and Spatial Sense

Spatial Sense

Spatial sense is the intuitive awareness of one's surroundings and the objects in them. Students need a rich learning environment that contains a variety of geometric objects that they can manipulate in order to discover the geometric properties of objects and the relationships among them.

Communication Skills

Students need to develop and use a variety of communication skills in geometry. Geometry uses a formal language to describe objects and their interrelationships and movements in space. In the Formative Years, however, teaching students the formal language of geometry is not as important as helping them to identify geometric properties and principles and to use pictures or everyday language to explain their observations. In the Transition Years, the student's personal understandings can grow into a correct and effective use of the vocabulary of geometry. The teacher's role in language development is to blend the student's own language with the correct mathematical terminology.

Visualization

Geometry is best learned through active exploration. Students discover relationships and develop their spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures. The ability to visualize is particularly important in the study of geometry. Students need to be able to draw images in their minds of how shapes look, and to be able to manipulate these images mentally.

Co-ordinate Geometry

The study of co-ordinate geometry in the Transition Years should include applications (e.g., the graphing of data from experiments and formulas) and also the formal mathematical treatment of co-ordinate geometry on the xy-plane. Students' experiences in graphing and interpreting should include both linear and non-linear relations. With respect to operations involving the xy-plane, by the end of Grade 9 students should have an understanding of the concept of a relation and of the relationship between the form of an equation and the shape of the corresponding graph. They should also have specific skills with respect to linear relations, such as graphing from an equation. Computers and graphics calculators are very powerful aids in studying co-ordinate geometry, and all students should have access to them.

Measurement

The Importance of Measurement

Understanding measurement is essential for all students. In practical terms, students need measurement skills to function in today's world. As well, many teaching models for other topics in mathematics assume that students possess basic measurement skills. Measurement concepts are important in other curriculum areas, such as science, environmental studies, and technology. Good instruction in measurement should not only teach skills, but also expand the student's ability to learn and understand mathematics.

Measurement Activities

Students must be actively involved in problem-solving activities if they are to learn about measurement. Understanding of measurement, both attributes and process, must be carefully developed through a variety of activities that allow students to compare, order, and then measure. In order to understand the origin and function of formulas, students need numerous opportunities to explore relationships informally by manipulating objects and models, and to identify generalizations from observed patterns. In the Formative Years, formulas should be proposed by students as the need arises, and should be expressed in their own language.

Attributes of Measurement

Types of measurement, such as length, area, capacity, and volume, need to be encountered on a continual basis in informal but authentic problem-solving settings. Undertaking measurement activities related to other topics in mathematics as well as other curricular areas will help students gain a greater insight into measurement attributes and concepts and develop an understanding of which units are appropriate for particular purposes.

Estimation

Because it is essential for students to learn and apply estimation strategies, estimation should be a natural part of all measurement activities, both in mathematics and in other curricular areas. A variety of estimation strategies should be taught, and students should come to understand that the accuracy of the estimate needed will vary according to the circumstances.

Patterning and Algebra

Patterns

Patterns are everywhere in our world. Patterns are the link students need to make connections between mathematics and the world in which they live. Exploring patterns leads students to develop mathematical competence and gain an appreciation for the beauty of mathematics.

It is essential for students in the Formative and Transition Years to explore patterns in order to develop understanding of the concept of a variable and of algebraic thinking.

Patterns in Geometry

In geometry, students need opportunities to build structures and study patterns in tiling, wallpaper, quilts, and furniture arrangements. Students can even form human patterns with their bodies in the gym or outdoors.

Patterns in Measurement

Measurement activities enable students to discover patterns, since to measure means to repeat a standard or non-standard unit with a scale or tool, and a formula is nothing more than a general statement about some observed pattern. Students need an environment that includes many opportunities to measure and to identify patterns.

Patterns in Number and Language

Number patterns are abundant in the student's world. Nature and our modern technological world provide many numerical patterns. The calculator can be used extensively to explore patterns involving repeated digits and skip-counting. Language is another rich source of patterns. Students can examine patterns in sounds, or words, or stories.

Algebraic Modelling

Algebraic modelling is a powerful method for solving problems. Students begin to learn about algebraic modelling in Grade 9 and continue throughout high school to develop the skills necessary to use it effectively. The foundations of algebraic modelling, however, are established much earlier. Through investigations involving patterns, including stating rules for patterns, students gradually develop the concept of a variable. In the Formative Years, students may use words to identify the rule for a pattern. By the Transition Years, students will use symbols to identify pattern rules, and will begin to write formulas that describe simple relationships represented graphically.

Teachers are expected to view algebraic modelling as one strategy amid a wide range of strategies and not leave students feeling that this is memorization work. If a given problem can be solved using other strategies besides algebraic modelling, the teacher should value all reasonable methods.

In Grade 9, formal treatments of algebra begin. Students must learn to solve linear equations and must be given opportunities to acquire the manipulation skills required to do so. Students' experiences with manipulation of algebraic expressions, however, should be limited to simple examples involving the basic operations necessary to solve equations.

Data Management and Probability

Real-World Connections

Mathematics programs must give significant attention to data management and probability. Graphing, statistics, and probability are closely linked, and activities involving their use are highly relevant to problem-solving tasks encountered in everyday life. Such activities provide students with the opportunity to think, use, understand, and interpret meanings of number – in other words, to develop number sense.

Activities involving data management and probability provide a meaningful context for the use of computational skills and offer students opportunities for representing, interpreting, and discussing information that applies to other mathematics concepts and skills. These activities help students to develop critical-thinking and problem-solving skills.

Probability

The study of probability in the Formative Years should be kept to an informal/incidental treatment of chance as it relates to games and simple everyday situations. Probability experiments and simulations should be developed in the Transition Years. Theoretical probability calculations should be left to the Specialization Years.

Applications

Graphing, statistics, and probability should not be treated in isolation, but used as tools to apply mathematics concepts and skills in meaningful contexts in all curriculum areas. Activities should emphasize collecting and interpreting data, formulating hypotheses, and devising procedures to test them. Activities may range from gathering statistics from projects, concrete graphs, calendars, and weather reports in the early grades to more complex probability experiments in the later grades. At all levels, however, activities must be concrete, manipulative, and based on students' experiences.

Outcomes from *Policies and Outcomes, Grades 1–9*

Grade 3

By the end of Grade 3, students will:

- begin to demonstrate attention to accuracy, thoroughness, persistence, and creativity in conducting an investigation;
- be able to follow the steps in a standard method of inquiry;
- identify more than one solution to a problem and show respect for other people's solutions;
- work constructively with others on projects;
- ask questions about the world around them and look for answers to these questions, working both alone and with others.

Grade 6

By the end of Grade 6, students will:

- conduct investigations with accuracy, thoroughness, persistence, creativity, and honesty;
- use different methods of inquiry to solve a variety of problems in mathematics, science, and technology;
- examine different explanations of and solutions to a problem, determine their validity, and apply the most appropriate solution ;
- work willingly alone or with others, as required by the project;
- investigate phenomena in their immediate environment and beyond and communicate their findings, working both alone and with others.

Grade 9

By the end of Grade 9, students will:

- analyse investigations to determine how accuracy, thoroughness, persistence, and creativity contribute to the solution of a problem;
- use a variety of methods of inquiry, including self-designed methods, and evaluate their effectiveness in solving various problems;
- solve problems in different ways and accept that other people's ways of solving them may be equally valid;
- explain why collaboration is important to the advancement of knowledge;
- use a variety of methods, technical devices, and processes to conduct inquiries at regional and global levels and to communicate their findings, working both alone and with others.

More specifically, students will:

- solve problems with confidence;
- persevere in problem-solving activities;
- apply inquiry methods to a variety of problem-solving situations;
- develop and apply strategies to solve a variety of problems;
- verify and interpret results on their own;
- relate solutions and problem-solving strategies to new problems;
- use co-operative skills in group problem-solving activities.

Level 1

Characteristics of Student Performance

With assistance the student applies a process used in problem solving. The student understands simple one-step problems, but requires assistance to choose a strategy and communicate results. The student uses technology as directed by the teacher. The student occasionally participates in group problem-solving activities.

Sample Performance Indicators

The student:

- will occasionally take risks when encouraged to do so;
- requires motivation from others to engage in problem-solving activities;
- requires assistance to understand the problem;
- selects from a limited number of strategies and needs assistance to try other strategies;
- gives an answer (correct or incorrect) with either no explanation of the solution or an incomplete explanation;
- requires assistance to determine the reasonableness of a solution and to verify the answer;
- requires assistance to participate in group problem-solving activities.

Standards of Performance

Level 2	Level 3	Level 4
The student applies a process used in problem solving. The student understands part of the problem, chooses from a limited number of strategies, and communicates a solution. The student may use technology as a tool in problem solving. The student perseveres and takes risks when encouraged to do so. The student participates in group problem-solving activities.	The student describes and applies a process used in problem solving. The student understands the problem, chooses from a variety of strategies, and is able to communicate a reasonable solution. The student uses technology effectively as a tool in problem solving. The student is self-motivated, perseveres with challenging activities, and takes risks independently. The student co-operates in group problem-solving activities.	The student applies a problem-solving model. The student interprets the problem, chooses a creative strategy, and communicates a solution using appropriate mathematical language. The student extends and invents problem-solving activities through the creative use of technology. The student seeks out challenges and motivates others to find solutions to problems.
In addition, the student:	In addition, the student:	In addition, the student:
– will take risks, when encouraged to do so;	– takes risks;	– encourages others to take risks;
– perseveres in problem-solving activities when encouraged to do so;	– perseveres in problem-solving activities;	– demonstrates a commitment to finding solutions;
– demonstrates a partial understanding of the problem;	– understands the problem;	– intuitively visualizes and interprets problems; – independently identifies and organizes important information, distinguishes between pertinent and non-pertinent information, and recognizes when additional information is required;
– selects from a variety of strategies, including those that require the use of technology, but needs assistance in choosing the appropriate one;	– decides on and develops appropriate strategies to solve problems; – applies familiar strategies to new problems;	– develops sophisticated strategies to solve problems;
– gives an answer and begins to elaborate upon incomplete explanations, with assistance;	– gives a correct answer with a complete explanation of all the steps used;	– uses mathematical language in a precise way to communicate solutions;
– judges the reasonableness of a solution and verifies the answer, when directed to do so or when questioned;	– judges the reasonableness of a solution and verifies the answer;	– continually monitors the reasonableness of the selected strategies and the progress towards a solution during problem solving;
– works co-operatively to solve problems.	– works co-operatively and makes a significant contribution to the group.	– identifies and draws on resources within and outside the group that can be used to solve problems.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 3, students will:

- apply the concept of place value and use whole numbers and simple fractions in a variety of practical applications, including problem solving and estimation;
- perform accurately the basic operations of addition, subtraction, multiplication, and division of whole numbers.

More specifically, students will:

- read and print numerals and number words;
- describe the relationship between counting, grouping, and place value;
- construct and describe place-value representations of whole numbers;
- construct, describe, and use representations of fractions and decimals (e.g., amounts of, money);
- use personal strategies to recall basic number facts;
- apply computational procedures involving addition and subtraction of whole numbers;
- represent, describe, and model multiplication and division concepts in problem-solving situations;
- use a variety of strategies in mental computation;
- use a variety of estimation strategies involving quantity and computation;
- describe when estimation is an appropriate strategy;
- use the calculator as a mathematical tool;
- describe how operations can be applied to solving problems;
- construct a variety of problems involving number operations.

Sample Assessment Activities

1. Demonstrate with base-10 blocks why 134 is thirteen tens and four ones.

2. Enter [+][9] in the calculator and begin pressing [=][=] . . . Record all your numbers in a table. What did you notice first? Find at least one interesting pattern and explore it. See if your pattern continues into three digits. Can you explain why the pattern works? What else did you notice? What would happen with numbers other than 9?

3. How many combinations of two numbers are there that add to 12. Explain your reasoning using counters, a table, a diagram, a calculator, or any other method you like.

Level 1

Characteristics of Student Performance

The student relies on concrete representations and requires assistance in performing basic operations. The student demonstrates a good understanding of counting and grouping principles, but has a limited understanding of place value. The student uses a given strategy in estimation and/or mental calculation, with assistance.

Sample Performance Indicators

The student:

- demonstrates an understanding of conservation of number in a real-life context;
- uses concrete materials to construct representations of two-digit numbers;
- represents numbers symbolically and in concrete form;
- names the number for a given concrete representation of a two-digit number;

- identifies simple fractions represented concretely, pictorially, and in written form;

- counts money using pennies, nickels, dimes, and quarters;

- recalls basic addition and subtraction facts with the assistance of concrete models (i.e., up to 9 + 9 and 18 – 9);
- adds and subtracts two- and three-digit numbers without regrouping;

- demonstrates multiplication as repeated addition;
- demonstrates division as repeated subtraction;

- continues to develop strategies for estimation and mental computation and uses them when directed to do so;

- demonstrates the use of the function keys on a simple four-function calculator;

- identifies some of the important elements in a problem.

Standards of Performance

Level 2	Level 3	Level 4
The student represents, regroups, and renames numbers. The student understands and remembers basic number facts and uses a variety of strategies to perform computations. With teacher direction, the student chooses an appropriate method of computation from among estimation, mental calculations, calculator, or paper-and-pencil procedures.	The student describes how counting, grouping, and place value are related to one another, using a variety of methods and objects. The student sees relationships among the operations and uses a variety of methods in performing them. The student uses estimation and mental calculation strategies and the calculator as a tool where appropriate. The student makes judgements about the reasonableness of computational results.	The student has an intuitive sense of number. The student has a sound understanding of our numeration system, and is able to represent his or her understanding of that system through the use of concrete materials. The student develops and applies a variety of strategies for estimation and mental calculation. The student uses these skills in combination to make judgements about the reasonableness of computational results and of proposed solutions.
In addition, the student:	In addition, the student:	In addition, the student:
– reads and prints numerals to 999 and number words to 100; – constructs number meanings through real-world experiences; – compares and orders whole numbers; – counts by 2s, 5s, and 10s up to and down from 100; – represents two- and three-digit numbers concretely and pictorially;	– names and renames quantities to 999 (e.g., 52 as 52 ones, 5 tens and 2 ones, fifty-two, etc.); – uses place value to order three-digit numbers; – describes the relationship between counting, grouping, and place value;	– applies place-value concepts to numbers beyond 999;
– represents simple fractions (with denominators of not more than 10) concretely, pictorially, and in written form; – compares two simple fractions with the same denominator using concrete materials;	– represents equivalent fractions using concrete materials; – illustrates improper fractions using concrete materials;	– describes improper fractions using concrete materials; – uses concrete materials to solve problems involving fractions;
– writes an amount of money using decimal notation;	– represents decimals to tenths using concrete materials; – adds and subtracts amounts of money using decimal notation;	– identifies decimal parts of a whole; – compares and orders decimal numbers;
– recalls basic addition and subtraction facts; – recalls basic multiplication facts up to 5 x 5;	– recalls basic division facts up to 25 ÷ 5; – uses personal strategies to recall basic number facts;	– applies basic facts in solving problems;
– adds and subtracts two- and three-digit numbers with and without regrouping;	– adds and subtracts two- and three-digit numbers with and without regrouping using a variety of methods (e.g., adding on, using patterns, etc.);	– adds and subtracts numbers with more than three digits;
– interprets and records multiplication and division sentences in a variety of ways;	– solves multiplication and division sentences using a variety of methods and materials and in problem-solving activities;	– applies relationships among the operations (+, –, x, ÷) where appropriate;
– recognizes when estimation is an appropriate strategy; – recognizes when a computation could be performed mentally;	– uses estimation strategies to solve problems and to check if results are reasonable; – explores, invents, and shares a variety of mental calculation strategies;	– develops and applies a variety of strategies for mental calculation and estimation;
– uses a calculator to explore number patterns;	– recognizes when the use of a calculator is appropriate; – uses a calculator in a variety of ways (e.g., to perform operations, examine number relationships, solve problems, etc.);	– recognizes the capabilities and limitations of a calculator;
– identifies and uses the basic operation required to solve a one-step problem.	– solves problems involving more than one operation; – constructs problems from given information.	– constructs and solves problems that demonstrate the basic operations.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 3, students will:

- investigate the attributes of two- and three-dimensional figures by constructing models of them;
- describe the results of sliding, flipping, and turning a variety of objects and shapes, using their knowledge of spatial relationships and the effects of motion geometry;
- use simple grid applications in conducting investigations and playing games.

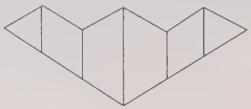
More specifically, students will:

- identify and sort two-dimensional and three-dimensional shapes and figures;
- construct two-dimensional and three-dimensional figures using a variety of concrete materials;
- construct a variety of three-dimensional structures and describe their properties;
- create and describe geometric patterns in two and three dimensions;
- recognize and model symmetry;
- explain movements that take an object and people from one position to another;
- solve problems in geometric contexts in a variety of ways (e.g., using computer software, concrete materials);
- use simple grid applications in games and investigations;
- relate geometric ideas to number and measurement ideas (e.g., base-10 materials to concepts of area and place value);
- recognize examples of geometric structures and designs in the world around them.

Sample Assessment Activities

1. Using plastic drinking straws and Plasticine, construct the tallest structure you can. Describe the changes you made to your design as you went along.

2. Construct a simple design using three or four pattern blocks. Using your design, make patterns showing flips, slides, and turns.



Level 1

Characteristics of Student Performance

The student uses language in a general way to express ideas about geometric objects and their relationships. With assistance, the student can produce written records of geometry activities. The student notices and can describe some ways of positioning objects in space. The student constructs common two- and three-dimensional figures and describes their properties, with assistance.

Sample Performance Indicators

The student:

- sorts a collection of items using a given criterion (e.g., number of sides, shape);
- builds a three-dimensional structure using a variety of materials and names the three-dimensional solids used;
- describes and continues two-dimensional patterns and names the basic shapes used;
- uses his or her own language to describe positions of objects in space;
- uses previous experiences to solve geometric problems, with assistance;
- describes the location of points on a grid, with assistance.

Standards of Performance

Level 2	Level 3	Level 4
The student uses correct geometric terminology to describe some specific properties and relationships of two- and three-dimensional shapes and figures. The student constructs two- and three-dimensional figures and begins to identify their properties. The student independently uses a given framework to make written records of geometry activities that involve manipulating or observing shapes in the environment. The student begins to notice and describe motions of objects in space.	The student uses correct geometric terminology and demonstrates understanding of the properties and relationships of two- and three-dimensional shapes and figures and of how objects move in space. The student explains geometric ideas clearly using a variety of methods. The student constructs a variety of two- and three-dimensional figures and objects and correctly describes their properties.	The student demonstrates understanding of the properties of complex two- and three-dimensional shapes and figures and of how objects move in space. The student uses technology in formulating and solving geometric problems.
In addition, the student:	In addition, the student:	In addition, the student:
– sorts and re-sorts a collection of items using his or her own stated criterion;	– groups objects using more than one criterion and records the results;	– generates and records intersecting sets (e.g., Venn diagrams, charts);
– describes similarities and differences in student-built three-dimensional structures (e.g., number of faces, edges, vertices);	– builds a three-dimensional structure and describes how symmetry, pattern, strength, and stability are reflected in it;	– uses knowledge of the characteristics of three-dimensional solids to solve a problem (e.g., constructing a bridge to support a given weight);
– creates two- and three-dimensional patterns using at least three shapes (e.g., cylinder, cube, triangular prism);	– creates two- and three-dimensional patterns using more than one criterion (e.g., repetition and symmetry) and records the results;	– creates complex geometric patterns in a variety of ways (e.g., using computer software);
– identifies slides, flips, and turns as motions of objects;	– demonstrates slides, flips, and turns using movements of objects and his or her own body;	– creates designs using combinations of slides, flips, and turns;
– uses previous experience in solving geometric problems;	– suggests possible ways to solve geometric problems;	– describes or demonstrates the explorations and decisions involved in solving geometric problems and the results achieved;
– describes location of points on a grid or map (e.g., A4).	– describes how to get from one point to another on a grid (e.g., two squares up followed by three squares to the right).	– draws a community map using an approximate scale.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 3, students will:

- estimate, measure, and record temperature, time, distance, length, perimeter, area, capacity and volume, mass, and amounts of money, using non-standard and standard (SI metric) units of measurement.

More specifically, students will:

- estimate to predict the results of a measurement activity;
- measure length, mass, capacity/volume, area, time, and temperature, using standard and non-standard units;
  select and use appropriate units and tools for measurement;
- record the results of measurement activities in a variety of ways;
- use measurement concepts to solve everyday problems (e.g., estimating the amount of money needed to make various purchases, making change).

Sample Assessment Activities

1. Draw a line about 15 cm long. Explain why you think your estimate is a good one.

2. Use a ruler to measure the following line.

3. Is 1 L a lot to drink at one time? Explain why you think it is or is not. How about 355 ml? or 10 ml?

Level 1

Characteristics of Student Performance

The student can measure using non-standard units, and identifies which tools are appropriate for a given measurement task. The student records results of measurement activities, with assistance.

Sample Performance Indicators

The student:

- compares at least three objects using general measurement terms (e.g., longest, heaviest);
- measures a given object using three or more non-standard units;
- recognizes which measurement tool is used for length, mass, temperature, time, capacity;
- reads time to the hour and half-hour (analog and digital);
- records results of a measurement task, with assistance.

Standards of Performance

Level 2	Level 3	Level 4
The student uses estimation to predict the results of measurement activities and begins to use standard units. The student uses a variety of methods to record results of measurement activities.	The student shows a sound understanding of the purpose of estimation and uses it effectively to predict the results of measurement activities. The student accurately uses the appropriate standard unit of measurement for a given task. The student makes an effort to represent differences in measurements accurately.	The student demonstrates a firm understanding of estimation and measurement concepts and applies them with a high degree of accuracy to solve everyday problems.
In addition, the student:	In addition, the student:	In addition, the student:
– distinguishes between making a "wild" guess and making an estimate based on past experience;	– uses previous estimates to predict the results of measurement activities as accurately as possible;	– uses comparisons with familiar objects in estimating measurements;
– explains the need for a common unit of measurement;	– estimates and measures length in metres and centimetres; mass in kilograms and grams; capacity in litres and millilitres; time in five-minute intervals; temperature in degrees Celsius, etc.;	– represents the relationship between a metre and a centimetre, etc.;
– compares the mass of various objects to one kilogram (capacity to one litre, etc.); – measures perimeter using non-standard and standard units; – recognizes the need for larger and smaller measurement units such as metre and centimetre;	– judges when to use larger or smaller measurement units (e.g., metres or centimetres); – measures and compares areas using non-standard units;	– creates a variety of shapes using a given length, area, or volume;
– records results of a measurement task independently.	– creates graphs, diagrams, etc., to report on a measurement activity.	– formulates measurement problems choosing appropriate units.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 3, students will:

- identify, extend, and create patterns and use these patterns to build models and solve everyday problems.

More specifically, students will:

- explore a wide variety of patterns in geometry, measurement, and number;
- use patterns to represent and solve problems;
- represent and describe mathematical relationships through patterning;
- use a variety of materials and media (e.g., software, stories, music) to create, describe, and extend patterns.

Sample Assessment Activities

1. In how many ways can you put together the other pattern blocks inside the hexagon? Record your answers below. How do you know you have all the ways?

2. Describe the patterns you see in this chart. You may wish to shade the patterns with different colours.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Level 1

Characteristics of Student Performance

The student recognizes, describes, and creates patterns using a variety of materials. The student uses a variety of mathematical symbols to represent number relationships.

Sample Performance Indicators

The student:

- recognizes and describes similarities and differences in shape, size, and quantity using concrete materials and real-world examples;
- recognizes, describes, and extends a pattern that is represented in concrete or pictorial form;
- recognizes number patterns;
- discovers number facts for addition (e.g., $5 = 0 + 5 = 1 + 4 = 2 + 3 = 3 + 2 \ldots$);
- recognizes signs (e.g., $+, -, >, <, =, \neq$) and uses them to denote mathematical relationships.

Standards of Performance

Level 2	Level 3	Level 4
The student creates and describes a variety of patterns and explains how pattern recognition can help in solving problems. The student represents relationships among numbers. The student uses patterns to construct number-fact families.	The student independently uses a variety of pattern structures to solve problems. The student correctly interprets the relationships among groups of numbers. The student begins to use this knowledge to predict missing terms in an equation.	The student demonstrates a highly developed sense of pattern and uses this knowledge with skill and imagination in designing and solving problems.
In addition, the student:	In addition, the student:	In addition, the student:
– recognizes, describes, extends, and creates a wide variety of repeating patterns (e.g., ABC, ABC, ...) using objects and numbers;	– uses concrete materials to copy, extend, and create geometric patterns; – recognizes, describes, extends, and creates a wide variety of growing patterns (e.g., A, AB, ABC, ... or ABC, ABBC, ABBBC, ...) using objects and numbers;	– develops repeated and growing number patterns of increasing complexity;
– explains how pattern recognition can be used to solve problems that have repetition as their theme; – explores number patterns using a calculator;	– creates numerical patterns by skip-counting, adding on, or subtracting and solves problems that contain such patterns; – identifies counting patterns on a hundreds chart;	– uses pattern recognition and pattern creation as strategies for solving problems;
– constructs number-fact families for addition and subtraction (e.g., $2 + 5 = 7$, $5 + 2 = 7$, $7 - 2 = 5$, $7 - 5 = 2$);	– applies knowledge of number-fact families to determine the missing term in a simple equation (e.g., $2 + ? = 7$);	– applies knowledge of number-fact families to a wide variety of problems;
– recognizes multiplication (x) and division (÷) signs and uses them to denote mathematical relationships; – recognizes the patterns of multiplication as repeated addition, and of division as repeated subtraction.	– represents the relationships between addition and multiplication and between subtraction and division in a given context.	– recognizes and applies his or her understanding of the relationship between multiplication and division when solving problems.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 3, students will:

- use a variety of methods to gather, analyse, display, and communicate information;
- understand simple ideas related to chance and probability and conduct investigations to explain them.

More specifically, students will:

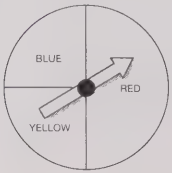
- collect and organize information related to familiar situations;
- display and record data;
- interpret and compare displayed information;
- apply data-management skills throughout the mathematics curriculum and in other subject areas;
- predict the probability that a particular event will occur;
- design and solve problems that involve collecting and analysing data.

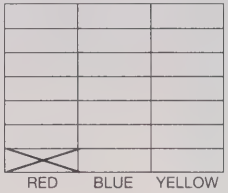
Sample Assessment Activities

1. When you spin the spinner, what colour is most likely to come up? Why do you think so?

   Spin the spinner twelve times and record your results below.

   If you kept spinning, what do you think the graph would look like?





2. Create a pictograph to display the following information. Have each picture stand for more than one person.

   Brown eyes: 20 people

   Green eyes: 4 people

   Blue eyes: 10 people

Level 1

Characteristics of Student Performance

The student displays and records data with assistance. The student makes predictions about situations based on personal experiences.

Sample Performance Indicators

The student:

- collects and organizes materials and/or information according to specific criteria;
- completes partially constructed tables or graphs;
- constructs and interprets concrete graphs and pictographs;
- answers specific questions about displayed data (e.g., bar graphs);
- makes predictions about situations based on personal experiences;
- uses appropriate vocabulary when discussing concepts of chance (e.g., often, sometimes, never; impossible, likely, probable).

Standards of Performance

Level 2	Level 3	Level 4
The student displays and records data in a prescribed way. The student interprets displayed information. The student begins to make reasoned predictions using data.	The student displays and records data in a variety of ways. The student analyses and compares displayed information. Through informal investigations, the student begins to link expectations that particular events will occur with simple notions of probability.	The student displays and records data in complex ways (e.g., in a circle graph) in cross-curricular contexts. The student draws conclusions from displayed information. The student uses concepts of chance to predict outcomes of events.
In addition, the student:	In addition, the student:	In addition, the student:
– decides collaboratively what information is to be collected (i.e., works with others to formulate an appropriate question);	– creates a relevant survey question;	– designs and conducts a survey or interview;
– gathers data by conducting a survey or interview and organizes this data;	– gathers data by conducting a survey or interview and organizes the data using several criteria;	– compares different ways of organizing data;
– constructs pictographs and bar graphs using a 1:1 scale;	– constructs pictographs and bar graphs using multiple scales (2, 5, 10);	– constructs different types of graphs and compares their suitability in various situations;
– interprets displayed information and lists results;	– analyses displayed information;	– discovers and verifies patterns in graphs; – draws conclusions from displayed information;
– uses concepts of chance and data from common experience to make reasonable predictions.	– predicts results of simple probability experiments (e.g., rolling a number cube, spinning a spinner) and/or games of chance, and tests the predictions by trying the experiments.	– predicts the probability that a particular event will occur and then provides reasonable data to support the prediction; – designs games or experiments using concepts of chance.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 6, students will:

- select appropriate calculation methods to solve problems with whole numbers, fractions, and decimals;
- demonstrate understanding of fractions and decimals and use them in a variety of practical applications, including problem solving and estimation.

More specifically, students will:
- formulate and solve a wide variety of single- and multi-step problems based on everyday situations using whole numbers, decimals, and fractions;
- demonstrate number sense in operating with whole numbers, decimals, and fractions;
- demonstrate an understanding of place value in operations, mental mathematics, and estimation;
- choose the most appropriate calculation method from among mental mathematics, estimation, calculator, or pencil-and-paper procedures, in problem-solving activities;
- use mental mathematics, estimation, calculators, and pencil-and-paper procedures involving whole numbers, decimals, and fractions, in problem-solving activities;
- demonstrate some understanding of per cent and ratio and rate.

Sample Assessment Activities

1. Solve the following mentally. Explain the strategy you use to solve each.

   a) $23 + 65$ d) $38 + 65$ g) $5800 \div 10$

   b) $38 + 62$ e) $328 \div 4$ h) 5×125

   c) $847 - 147$ f) $50 \times 6 \times 5$ i) $\$30.00 \times \$11.90$

2. Estimate the total of $\$6.15 + \$7.98 + \$8.11 + \$3.29$

   Explain your answer.

3. If carpet material costs $\$15.95$ m^2, what is the cost of carpeting a room that is 4.5 m by 5.2 m?

Level 1

Characteristics of Student Performance

The student demonstrates a limited understanding of place value and carries out some computational procedures. The student solves problems, estimates, and uses the calculator appropriately, with teacher assistance.

Sample Performance Indicators

The student:

- solves some one-step problems;
- demonstrates a limited understanding of the effect of operations on whole numbers;
- demonstrates a limited understanding of place value but has difficulty in applying it to reading and writing numbers and in estimation and calculation;
- recalls some multiplication and division facts to 100;
- demonstrates a limited understanding of pencil-and-paper procedures involving whole numbers and decimals;
- demonstrates a limited understanding of basic concepts involving fractions, such as representation, magnitude, order, and equivalence;
- demonstrates understanding of basic procedures in mental calculation and estimation, with assistance;
- uses a calculator to support the learning and application of basic facts, pencil-and-paper procedures, and skills in mental mathematics;
- calculates some simple rates and interprets some scale maps and diagrams;
- demonstrates a limited ability to represent per cent in relation to concrete models.

Standards of Performance

Level 2

The student uses mental calculation, estimation, calculators, and pencil-and-paper procedures involving whole numbers, decimals, and fractions to solve one-step problems. The student demonstrates a limited understanding of per cent and represents fractions in decimal form.

In addition, the student:

- solves one-step problems with whole numbers, decimals, and fractions;
- explains and applies basic properties of operations on numbers (e.g., 2 + 3 = 3 + 2; [6 x 4] x 5 = 6 x [4 x 5]);
- demonstrates understanding of the effects of operations on whole numbers (e.g., 65 ÷ 5 produces a value less than 65, but 65 x 5 produces a value greater than 65);
- demonstrates understanding of place value, but requires assistance in applying it consistently in reading and writing numbers and in estimation and calculation;
- recalls multiplication and division facts to 100;
- uses pencil-and-paper procedures involving whole numbers and decimals with inconsistent accuracy;
- represents fractions in decimal form;
- adds and subtracts fractions with simple denominators and multiplies a fraction by a whole number;
- uses concrete models to demonstrate understanding of basic concepts involving fractions, such as magnitude, order, and equivalence;
- uses mental calculation with inconsistent accuracy;
- uses estimation by rounding with varying degrees of success;
- uses a calculator effectively in applications and problem solving throughout the curriculum and in everyday life;
- calculates a variety of rates drawn from real life experiences (e.g., heart rate, walking speed);
- represents per cents in relation to concrete models.

Level 3

The student demonstrates understanding of the properties of operations related to whole numbers, decimals, and fractions. The student demonstrates a limited understanding of ratio and rate. The student chooses the most appropriate calculation method from among mental calculation, estimation, calculator, or paper-and-pencil procedures. The student solves a variety of two-step problems. The student demonstrates an understanding of place value in decimals.

In addition, the student:

- solves two-step problems using whole numbers, decimals, and fractions;
- applies basic properties of operations on numbers in calculation and estimation (e.g., 130 + 56 + 70 is simplified as [130 + 70] + 56; 5 x 9 x 2 is simplified as [5 x 2] x 9);
- demonstrates understanding of the effects of operations on whole numbers and decimals (e.g., 65 ÷ 0.5 produces a number larger than 65);
- demonstrates understanding of place value and uses it effectively in reading and writing numbers and in estimation and calculation;
- quickly recalls multiplication and division facts to 100;
- is proficient in the use of pencil-and-paper procedures involving whole numbers and decimals (e.g., can add up to three four-digit numbers, subtract two four-digit numbers, multiply a two-digit number by a two-digit number, divide by a one-digit divisor);
- demonstrates an understanding of basic concepts involving fractions, such as magnitude, order, and equivalence;
- adds and subtracts fractions with simple denominators, and multiplies a fraction by a whole number;
- effectively uses mental calculation (e.g., can add, subtract, multiply, and divide numbers ending in zeros);
- effectively uses a variety of estimation strategies;
- uses a calculator to investigate concepts in mathematics;
- chooses the most appropriate calculation method from among mental mathematics, estimation, calculator, or paper-and-pencil procedures to solve a variety of problems;
- applies a ratio to interpret maps and scale diagrams;
- represents per cents as parts of a whole (e.g., sketches 50 per cent on a pie chart).

Level 4

The student solves a variety of multi-step problems using appropriate computational strategies. The student explains place value, estimation, and computations involving whole numbers, decimals, fractions, and per cents.

In addition, the student:

- solves multi-step problems using whole numbers, decimals, and fractions;
- skilfully performs operations with whole numbers, decimals, and fractions;
- explains the relationships between place values (e.g., 0.23 is 2 tenths and 3 hundredths or 23 hundredths) and analyses problems involving estimation and calculation;
- applies arithmetic facts in solving problems;
- analyses a variety of methods of computation involving whole numbers, fractions, and decimals;
- experiments with a variety of operations, both simple and complex, involving fractions;
- evaluates a variety of mental calculation and estimation strategies;
- uses a calculator with skill and imagination to carry out independent explorations of concepts and relationships in mathematics;
- explains the relationship between ratio, rate, and fractions;
- solves per cent problems using a variety of methods and materials.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 6, students will:

- describe, make models of, and construct two- and three-dimensional figures and make comparisons among them;
- use co-ordinate systems to describe locations of points and figures;
- use their knowledge of motion geometry to identify the properties of figures and to create patterns and designs.

More specifically, students will:

- measure and construct angles using a protractor;
- identify, describe, and classify two-dimensional and three-dimensional figures;
- identify symmetry in two-dimensional and three-dimensional figures;
- recognize alternative ways of classifying shapes;
- use co-ordinate systems to describe points and figures;
- apply slides, flips, and turns in a variety of ways;
- pose and solve problems in a geometric context;
- examine three-dimensional models from different perspectives;
- identify intersecting, parallel, and perpendicular lines and use them in a variety of geometry activities.

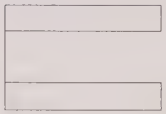
Sample Assessment Activities

1. Use pattern blocks to complete this design so that it has two lines of symmetry. Draw in the lines of symmetry.

2. Draw a series of at least five triangles. After the first triangle, each new one should be different in some way from those already drawn. Write down why you think each is different.

   How many different ways can two triangles be different?

3. Write a LOGO program to produce the following figure:



Level 1

Characteristics of Student Performance

The student uses the language of geometry loosely to describe the world. The student is beginning to make distinctions among members of classes of figures and objects but is not able to describe the differences clearly. The student demonstrates some understanding of the effect of motion in geometry.

Sample Performance Indicators

The student:

- selects a protractor when asked to measure or construct angles;
- recognizes and names common two- and three-dimensional figures in a variety of contexts (e.g., square, rectangle, triangle, cube, prism, cylinder, etc.);
- constructs common three-dimensional objects from a drawing using appropriate materials;
- identifies mirror symmetry in two- and three-dimensional figures;
- classifies shapes according to his or her own criteria;
- reads simple maps and plays co-ordinate games (e.g., Battleship) with assistance;
- identifies congruent figures by putting one object on top of another;
- uses slides, flips, and turns to make designs with concrete materials;
- given a shape and its image under a single motion, identifies the motion;
- recognizes and describes the occurrence or application of geometric properties and principles in the real world and in other subjects.

Standards of Performance

Level 2

The student uses the language of geometry appropriately to describe the world and makes distinctions in classifying figures and objects. The student creates and describes two- and three-dimensional figures. The student uses geometrical concepts to solve problems. The student demonstrates an understanding of the effect of motion in geometry.

In addition, the student:

- uses a protractor to measure and construct angles;
- describes the properties of two- and three-dimensional figures using appropriate language;
- classifies angles by size;
- constructs three-dimensional objects from given descriptions;
- creates two- and three-dimensional figures and objects that have mirror symmetry;
- classifies shapes according to given criteria;
- reads simple maps and plays simple co-ordinate games;
- identifies congruent figures by measuring side length and angle size;
- uses slides, flips, and turns to construct patterns and designs in a variety of ways;
- recognizes slides, flips, and turns in patterns and designs in the environment;
- solves problems in geometric contexts that involve the direct application of learned concepts and skills.

Level 3

The student uses the language of geometry effectively to describe shapes in the environment and to discuss geometric ideas. The student analyses two- and three-dimensional figures to solve geometric problems. The student demonstrates understanding of the effects of motion in geometry and creates related patterns and designs.

In addition, the student:

- estimates angle size within a reasonable range;
- classifies two-dimensional figures according to angle and side properties;
- classifies three-dimensional figures according to the number of faces, edges, and vertices;
- recognizes two-dimensional nets of three-dimensional solids;
- describes rotational symmetry;
- proves that a two-dimensional figure has mirror symmetry (e.g., using paper folding, MIRA);
- classifies shapes according to several given criteria;
- plots points in a co-ordinate system;
- proves figures are congruent (e.g., using paper folding, MIRA, and various computer-software packages);
- uses computer programs and languages such as LOGO to explore and apply slides, flips, and turns;
- visualizes and describes the effects of slides, flips, and turns, singly and in combinations;
- solves problems in geometric contexts that involve seeing relationships between or among concepts and/or skills.

Level 4

The student demonstrates a firm understanding of the language of geometry and independently generates novel applications of learned geometric concepts.

In addition, the student:

- uses angle size to define direction (e.g., N45°W);
- describes the properties of irregular two- and three-dimensional figures;
- designs two-dimensional nets of three-dimensional solids;
- uses understanding of symmetry in other areas of the curriculum (e.g., art, design, technology);
- classifies shapes according to a variety of criteria;
- uses the co-ordinate system in problem-solving activities;
- informally proves two figures are congruent by measurement;
- devises instructions for a computer to produce desired shapes and paths;
- applies knowledge of slides, flips, and turns to projects in other subject areas (e.g., art and music composition);
- generates alternative solutions to problems in geometric contexts.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 6, students will:

- estimate, measure and/or calculate, and record temperature, time, length, perimeter, area, capacity and volume, mass, amounts of money, distance, and speed, using appropriate units of measurement.

More specifically, students will:

- use a variety of tools to estimate and measure length, perimeter, area, capacity, volume, mass, a period of time, and temperature using standard units in problem-solving activities;
- use comparisons with familiar objects in measurement activities;
- explain the differences between estimation and precise measurement;
- apply simple measurement formulas to solve problems;
- use an understanding of the relationship among various metric units to compare measurements of objects;
- use measurement skills in other areas of the curriculum and to solve everyday problems.

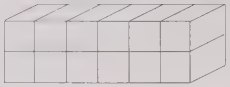
Sample Assessment Activities

1. Find parts of your body that are approximately 1 cm long, 1 dm long, 1 m long.

   Use these to estimate the measurements of other parts of your body (e.g., height, head circumference, etc.)

   Record your findings in chart form.

2. How many different rectangular solids can you build with twelve cubes? Compare the volume of each object to its surface area. Write a report, with diagrams, to display your results.



Level 1

Characteristics of Student Performance

The student uses appropriate instruments to measure length, capacity, and mass using a variety of units. The student estimates using given referents and compares the difference between estimates and measurements. The student applies simple formulas in measurement activities, with teacher assistance.

Sample Performance Indicators

The student:

- measures length, perimeter, area, capacity, mass, and temperature, using given tools and a variety of units, with assistance;
- identifies the relationship between the twelve-hour clock and the twenty-four-hour clock and reads both analog and digital time devices;
- uses comparisons with given objects in estimating measurements (e.g., a desk is about one metre high);
- identifies the difference between estimation and measurement, with assistance;
- estimates and measures the perimeter and area of irregular figures using non-standard units, with assistance;
- performs a required calculation when given the appropriate formula, with assistance;
- recalls the relationship between common units (e.g., metres and centimetres, litres and millilitres, grams and kilograms).

Standards of Performance

Level 2	Level 3	Level 4
The student measures accurately using standard units and appropriate methods (e.g., estimation, formulas). The student solves problems that require the use of learned measurement skills.	The student applies measurement concepts and skills correctly and with understanding to solve a wide variety of practical problems. The student sees the value of estimation and makes comparisons with familiar objects in making reasonable estimates.	The student estimates effectively, and routinely applies an understanding of measurement in all areas of the curriculum and in everyday life. The student develops formulas and procedures for determining measurements in problem-solving activities.
In addition, the student:	In addition, the student:	In addition, the student:
– chooses appropriate tools and measures with precision length, perimeter, area, volume, capacity, mass, and temperature using prescribed units;	– measures with precision and chooses the appropriate units and tools for the task;	– evaluates the effectiveness and precision of various measuring tools and the appropriateness of various units for different types of measurement tasks;
– makes comparisons with familiar objects in estimating measurements;	– determines the relative usefulness of many comparisons with familiar objects used in estimation activities;	– makes appropriate comparisons in estimating area and volume using appropriate units;
– identifies the difference between estimation and precise measurement;	– judges the reasonableness of the estimate in measurement activities;	– evaluates the effectiveness of a variety of strategies for estimating measurements;
– estimates and measures the perimeter and area of irregular figures using non-standard units and a variety of tools;	– estimates and measures the perimeter and area of irregular figures using standard units and a variety of tools;	– determines the relative usefulness of methods for estimating and measuring the perimeter and area of irregular figures;
– uses correct units and performs a required calculation, when given the required formula;	– develops formulas for determining perimeter and area (e.g., of rectangles, squares, and triangles) through a variety of measurement activities;	– develops formulas for determining the perimeter and area of parallelograms and simple prisms, with assistance;
– uses the prefixes in the metric system correctly.	– makes simple metric conversions (e.g., millilitre to litre; milligram to gram; metre to kilometre).	– makes complex metric conversions (e.g., millilitre to kilometre).

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 6, students will:

- investigate and explain the relationships among patterns in mathematics and in natural and human-made environments.

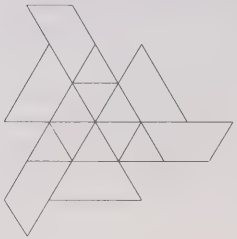
More specifically, students will:

- identify, extend, and create patterns in geometry;
- identify, extend, and create number patterns involving whole numbers and decimals;
- describe patterns;
- solve problems by applying a patterning strategy;
- recognize and identify patterns in material from other subject areas and in the world around them;
- identify formulas as generalizations about observed patterns.

Sample Assessment Activities

1.

   Continue the above pattern. How many dots would be in the twentieth rectangle? How would you state a general formula describing the pattern?

2. 

   If is worth 1, what is the value of the above design.

   If is worth 3, what is your design worth?

Level 1

Characteristics of Student Performance

The student uses concrete materials to assist in recognizing and creating patterns. The student shows some ability to describe patterns, with assistance. The student creates number patterns and records data in tables and graphs.

Sample Performance Indicators

The student:

- uses concrete materials to create geometric patterns;
- recognizes patterns but requires assistance to describe them;
- creates number patterns involving whole numbers;
- identifies patterns in the surrounding world;
- records data in tables and graphs.

Standards of Performance

Level 2	Level 3	Level 4
The student recognizes some rules for making patterns, and can apply them with assistance. The student uses a variety of methods and materials to record patterns and draw conclusions from them.	The student uses a repertoire of strategies and rules for patterning. The student demonstrates skill in identifying patterns and describes them effectively using mathematical language.	The student develops generalizations about a wide variety of patterns and represents them symbolically.
In addition, the student:	In addition, the student:	In addition, the student:
– identifies and extends geometric patterns;	– uses geometric patterns to explain numeric patterns (e.g., uses base-10 or other materials to represent 0.25, 0.50, 0.75, ...);	– identifies numeric patterns in geometric patterns;
– uses his or her own words to describe a pattern;	– uses appropriate mathematical language to describe patterns; – records conclusions involving patterns using mathematical formats such as tables and graphs;	– represents generalizations about observed patterns symbolically;
– identifies, extends, and creates number patterns involving whole numbers and decimals;	– uses patterns involving whole numbers and decimals to solve problems;	– reconstructs mathematical patterns in material from other subject areas and in the surrounding world;
– describes patterns in material from other subject areas (e.g., art, the social sciences, physical education) and in the surrounding world;	– solves problems in other subject areas and the surrounding world by applying a patterning strategy;	– interprets and analyses mathematical patterns in material from other subject areas and in the surrounding world;
– records conclusions involving patterns using tables and graphs.	– identifies formulas from tables and graphs.	– develops formulas as generalizations about observed patterns.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 6, students will:

- use a variety of technologies to gather, analyse, interpret, evaluate, and display data;
- investigate how chance and probability operate in a variety of situations and explain their findings.

More specifically, students will:

- collect, organize, describe, read, and interpret data using a variety of methods and tools;
- discuss chance and probability in everyday situations;
- describe the chances of winning in games and the probability that particular events will occur in experiments;
- apply data-management skills and skills in determining probability to other areas of the curriculum.

Sample Assessment Activities

1. What size of eggs is the best buy? Investigate.

2. Ann has three sweatshirts, four pairs of shorts, and two pairs of shoes. How many different outfits can she wear? Illustrate them below. If one sweatshirt is blue, approximately what per cent of the possible outfits will have a blue sweatshirt?

3. Which letters of the alphabet are used most frequently? Which are used the least? Investigate and present your report in written form.

Level 1

Characteristics of Student Performance

The student collects, organizes, and graphs data, with assistance. The student can read data from tables and graphs. The student recognizes elements of chance in everyday occurrences.

Sample Performance Indicators

The student:

- collects and organizes data for a given question, with assistance;

- reads and constructs bar graphs, pictographs, and broken-line graphs in which the scale represents a one-to-one correspondence (scale in units);

- perceives success in board/card games as the result of luck rather than of strategy based on knowledge of probability;
- recognizes certainty and impossibility and that some events are more likely to occur than others.

Standards of Performance

Level 2	Level 3	Level 4
The student collects and organizes data systematically and represents them in an appropriate way. The student interprets data in order to answer questions. The student can identify some external factors that influence chance in everyday situations.	The student collects, organizes, and displays data in a variety of ways. The student interprets data represented in graphs. The student considers how external factors influence the operation of chance in simple everyday situations.	The student shows independence and initiative in conducting data-management activities in real-life situations. The student makes sophisticated comparisons of data, including data beyond the direct scope of the question. The student independently considers the impact of many types of external factors on the operation of chance.
In addition, the student:	In addition, the student:	In addition, the student:
– carries out a survey to collect data; – designs methods to organize collected data; – predicts the response to a survey question before carrying out the survey;	– independently carries out a survey and organizes the data gathered; – writes a description of the results of a survey;	– critically evaluates the process used in a data-management activity (e.g., questions the effect on survey results of limited sample size); – compares information about the same question gathered from different sources;
– explains the meaning of terms related to surveys and graphs (e.g., "sample", "bar graph"); – reads, interprets, and constructs graphs in which the scale represents a many-to-one correspondence (e.g., scale in 10s or 100s);	– uses the language of surveys and graphs effectively to discuss methods and findings; – selects the appropriate type of graph for a given set of data (e.g., bar graph, pictograph, broken-line graph, circle graph);	– evaluates arguments that are based on analyses of data; – makes inferences based on analyses of data and develops convincing arguments to support them;
– can occasionally predict that some events will occur more frequently than others (e.g., in board/card games); – correctly interprets commonly used statements about chance (e.g., a fifty-fifty chance, odds of three to one, the prediction possibilities of a batting average); – predicts the possible outcomes of a situation and explains his or her reasoning (e.g., makes a prediction about the best time to go to a restaurant).	– makes decisions based on observation of the frequency of occurrence of events (e.g., in number cubes/board/card games); – discusses in realistic terms the chances of winning in draws and games of chance, such as lotteries, raffles, bingo.	– accurately assesses probability in relatively complex games of chance and everyday situations.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 9, students will:

- evaluate the effectiveness of various calculation methods for solving different types of problems;
- demonstrate understanding of per cents, integers, ratios and rates, powers, and square roots, and use them in a variety of practical applications, including problem solving and estimation.

More specifically, students will:

- use appropriate methods of computation involving whole numbers, decimals, per cents, integers, ratio and rate, powers, and square roots in applications, problem solving, and problem posing;
- use estimation and mental calculation to make decisions about situations in everyday life;
- determine the reasonableness and accuracy of numeric information from a variety of sources;
- decide whether an estimate or an exact answer is required in a problem-solving activity;
- use calculators as tools for calculation and to discover mathematical concepts;
- explain integers, ratio and rate, per cent, rationals, powers, and roots and use them to solve practical and mathematical problems.

Sample Assessment Activities

1. The regular price of an audio player is $449.57. How much will you save if you buy it now at 25 per cent off?

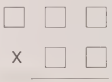
2. For each of the following estimates, state which answer makes sense. For those that do not make sense, state a more reasonable estimate.

Cost	Estimate
5 CD's for $14.99 each	$75.00
15 kg of sugar @ $0.79/kg	$21.00
9 litres of milk @ $1.25/l	$11.00
25 light bulbs @ $4.95 for 4	$20.00

3.

Name the number that each letter could represent. Explain.

4. Using the digits 6, 7, 8, 9, 0, how would you arrange the digits to produce:
   a) the largest possible product?
   b) the smallest possible product?



Level 1

Characteristics of Student Performance

With assistance, the student performs some operations involving estimation, mental calculation, use of a calculator, and calculations involving whole numbers, decimals, fractions, and per cents. The student demonstrates a limited understanding of ratio and rate.

Sample Performance Indicators

The student:

- determines the types of calculation called for in an application, with assistance;
- determines whether an estimate or an exact answer is appropriate in a given application, with assistance;
- estimates quantity and computations by rounding;
- uses given methods of mental calculation, with assistance;
- strings together more than one operation on a calculator;
- performs calculations involving whole numbers, decimals, fractions, and per cents, with assistance;
- applies ratio and rate and per cent in problem-solving activities, with assistance.

Standards of Performance

Level 2	Level 3	Level 4
The student uses computational estimation strategies to determine the reasonableness of an answer in problem-solving activities. The student chooses an appropriate method of calculation to solve a variety of problems, but is not consistently correct in his or her calculations. The student uses integers, ratio and rate, per cents, powers, and square roots in problem-solving activities.	The student chooses from a variety of estimation, mental mathematics, and calculation methods and strategies to perform a variety of problem-solving tasks. The student demonstrates proficiency in calculations and understands and uses integers, ratio and rate, per cents, rationals, powers, and square roots in problem-solving activities.	The student analyses a range of methods and strategies that can be used in estimation, mental mathematics, and calculation and develops new ones. The student applies basic concepts used in number theory to solve practical and mathematical problems.
In addition, the student:	In addition, the student:	In addition, the student:
– chooses a method of calculation to solve a problem;	– chooses from a variety of calculation methods to solve a problem;	– evaluates a variety of methods of calculation used in problem-solving activities;
– uses computational estimation strategies when asked to determine whether an answer appears to be correct; – determines whether an estimate or an exact answer is appropriate in a given application;	– uses estimation strategies to check the reasonableness of the results of a calculation (including results obtained with a calculator);	– evaluates a variety of computational estimation strategies;
– uses prescribed methods to perform mental calculations;	– uses a variety of methods of mental calculation;	– evaluates a variety of methods of mental calculation;
– uses the calculator to determine number relationships and patterns;	– uses a calculator correctly and efficiently for calculation and during investigations;	– investigates and explains the use of the many function keys on a scientific calculator;
– is correct most of the time in calculations involving whole numbers, decimals, rationals, per cents, integers, ratio and rate, powers, and square roots;	– demonstrates proficiency in calculations involving whole numbers, decimals, rationals, per cents, integers, ratios and rate, powers, and square roots in problem-solving activities;	– analyses methods of computation and develops new ones;
– uses integers, ratio and rate, per cents, rationals, powers, and square roots in problem-solving activities.	– uses integers, ratio and rate, per cents, rationals, powers, and square roots appropriately and correctly in problem-solving activities.	– uses a variety of methods and tools to explain the interrelationships among whole numbers, fractions, decimals, per cents, integers, ratio and rate, powers, and square roots; – applies concepts used in number theory to solve practical and mathematical problems.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 9, students will:

- solve problems in transformational geometry;
- use the properties of two- and three-dimensional figures in a variety of applications in mathematics, science, and technology;
- use co-ordinate geometry to describe, interpret, and represent relationships defined by equations and formulae.

More specifically, students will:

- complete basic construction tasks using a variety of tools and techniques;
- identify and describe properties of two-dimensional shapes such as congruent parts, line symmetry, rotational symmetry, angle properties and use them in problem-solving activities;
- identify angle properties of intersecting, parallel, and perpendicular lines and use them in problem-solving activities;
- use a variety of methods, tools, and materials to identify and represent properties of translations, reflections, rotations, and dilatations;
- use co-ordinate geometry to describe, interpret, and represent relationships defined by equations and formulae;
- use geometry to create patterns and describe the physical world.

Sample Assessment Activities

1. Draw any triangle. Choose three measurements of angles and/or lengths of sides, and use only these to tell a partner how to draw a triangle that is congruent to yours. What combinations of angles and sides will work?

2. How many ruler and compass constructions can also be performed using a MIRA (e.g., bisecting an angle)? Can the MIRA be used to perform constructions that cannot be constructed using a ruler and compass?

Level 1

Characteristics of Student Performance

The student identifies, sketches, and builds common geometric figures and objects found in the environment. The student identifies basic geometric properties and understands the concept of tiling the plane. The student uses concrete materials to demonstrate an understanding of motion geometry. The student requires assistance in basic geometric constructions.

Sample Performance Indicators

The student:

- completes construction tasks in two or three dimensions (e.g., bisects an angle, builds a three-dimensional model from a net), when given detailed direction;
- identifies geometric figures;
- uses available geometry-related programs on the computer,. with assistance;
- demonstrates an understanding of slides, flips, and turns using concrete materials;
- identifies angle properties of intersecting, parallel, and perpendicular lines by direct measurement;
- identifies figures that will tile a plane;
- sketches and builds three-dimensional figures and objects, with assistance.

Standards of Performance

Level 2	Level 3	Level 4
The student classifies geometric figures and describes their properties. The student creates tiling patterns and completes basic geometric constructions. The student sketches three-dimensional figures and demonstrates an understanding of motion geometry.	The student applies geometric concepts in a variety of problem-solving activities. The student describes these concepts using appropriate mathematical language.	The student formulates and analyses problems using geometric concepts.
In addition, the student:	In addition, the student:	In addition, the student:
– completes construction tasks in two or three dimensions using a variety of tools and techniques, with some direction (e.g., bisects an angle, draws a perpendicular to a line using paper folding, MIRA, computer, ruler and compass);	– completes construction tasks in two or three dimensions accurately;	– explains possible alternative methods of construction;
– explains geometric terms and concepts; – identifies and classifies geometric figures and describes their properties in his or her own language;	– identifies geometric properties and concepts and describes/explains them in precise mathematical language;	– analyses geometric properties and concepts and uses precise mathematical language to communicate his or her findings;
– uses available geometry-related programs on the computer, with assistance;	– uses geometry-related programs on the computer;	– predicts geometric properties using geometry-related programs on the computer;
– identifies slides, flips, turns, and dilatations of two-dimensional shapes;	– represents and describes slides, flips, and turns of two-dimensional shapes in a variety of ways (e.g., as mappings);	– describes transformations as mappings on a Cartesian plane;
– identifies angle properties of intersecting, parallel, and perpendicular lines;	– uses the angle properties of intersecting, parallel, and perpendicular lines to solve problems;	– investigates a variety of angle properties and describes his or her findings;
– creates regular figures that will tile a plane;	– creates tessellations of the plane with given symmetries and patterns;	– creates tessellations of the plane, identifying the transformations used in the design;
– sketches three-dimensional figures on isometric dot paper.	– builds models from drawings of top, side, and front views of three-dimensional figures; – sketches top, side, and front views from three-dimensional objects.	– creates, designs, and analyses three-dimensional figures using a variety of methods and tools (e.g., computer software).

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 9, students will:

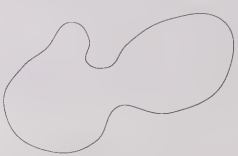
- identify, develop, and apply procedures for a variety of measurement activities, using concrete materials and appropriate units of measurement.

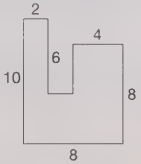
More specifically, students will:

- use comparisons with familiar objects in a variety of measurement activities; determine when measurement or estimation is more appropriate; estimate and measure length, perimeter, area, volume, angles, capacity, and mass in problem-solving activities using appropriate strategies, units, and tools;
- explain the concepts of length, area, volume, capacity, mass, temperature, and time;
- use an understanding of the relationship among common metric units (e.g. mm/cm, cm/mm) in measurement activities;
- construct regular and irregular figures using a variety of materials;
- estimate and measure the perimeters and areas of irregular figures, using a variety of techniques.

Sample Assessment Activities

1. How would you determine the areas of these shapes?





2. Figure A is a stack of playing cards. Figure B is the same pile of cards, only slanted to the side. Which figure has greater volume? Why?

Figure A Figure B

Level 1

Characteristics of Student Performance

The student demonstrates some ability to estimate measurements and uses appropriate units with assistance. The student uses simple formulas to solve problems. The student measures and constructs two-dimensional figures.

Sample Performance Indicators

The student:

- uses comparisons with given objects when estimating;
- selects appropriate units, with assistance;
- performs a required calculation when given the appropriate formula;
- constructs or illustrates regular two-dimensional figures with a given perimeter, with assistance;
- measures the perimeter of irregular two-dimensional figures, with assistance.

Standards of Performance

Level 2	Level 3	Level 4
The student employs a number of estimation strategies in measurement activities using appropriate units. The student applies formulas in measurement activities. The student solves a variety of problems involving two-dimensional figures.	The student analyses estimation strategies, selects one that is suitable, and applies it using appropriate units. The student constructs a variety of two- and three-dimensional figures and develops and applies formulas for them.	The student evaluates various measurement methods to determine which is the most precise. The student constructs a variety of complex two- and three-dimensional figures and develops and applies formulas for these figures.
In addition, the student:	In addition, the student:	In addition, the student:
– uses comparisons with familiar objects when estimating; – judges the reasonableness of an estimate, with assistance;	– recognizes when measurement or estimation is more appropriate; – estimates, and judges the reasonableness of the estimate, using appropriate units;	– evaluates various strategies for estimating measurements to determine which is the most precise;
– selects the appropriate unit of measurement;	– selects appropriate units and converts from one unit to another with accuracy;	– explains the relationship among various units of measure (e.g., $cm^2 \Rightarrow m^2$, $dm^2 \Rightarrow l$);
– applies formulas to determine the perimeter and area of rectangles, triangles, parallelograms, and trapezoids;	– develops and applies formulas for the circumference of a circle, area of a parallelogram, trapezoid, and circle; surface area of a prism, pyramid, and cylinder; volume of a prism;	– develops and applies formulas for irregular figures;
– constructs or illustrates regular two-dimensional figures with a given perimeter using a variety of materials and tools;	– constructs regular two- and three-dimensional figures with a given area/volume;	– constructs irregular two- and three-dimensional figures with a given area/volume;
– estimates and measures the perimeter of irregular two-dimensional figures (e.g., a racetrack).	– estimates and measures the area of irregular two-dimensional figures.	– estimates and measures the volume of irregular three-dimensional figures.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 9, students will:

- use algebraic notation and modelling in solving problems.

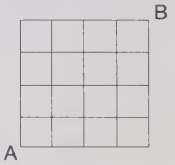
More specifically, students will:

- express generalizations about patterns both in words and algebraically, and develop formulas as generalizations about observed patterns;
- simplify algebraic expressions using a variety of methods and tools;
- solve linear equations in one variable using a variety of methods and tools;
- solve problems using linear equations and inequalities;
- use patterns, tables, and graphs to represent and solve problems;
- apply the concepts of a variable, an expression, and an equation;
- use calculators and computers to investigate patterns;
- graph relations that are relevant to other subject disciplines.

Sample Assessment Activities

1. If 2^{100} is expanded, what is the digit in the units place?

2. How many paths are there from point A to point B?
(You can move up or to the right only).



Level 1

Characteristics of Student Performance

The student simplifies algebraic expressions using concrete materials, with assistance. The student applies patterning and algebra skills, with assistance.

Sample Performance Indicators

The student:

- extends a variety of patterns derived from number and geometric concepts using a wide variety of materials;
- simplifies algebraic expressions using concrete materials, with assistance;
- solves problems and linear equations involving one operation using trial and error, inspection, and concrete materials;
- completes a table and graph showing information from an experiment or pattern, with assistance;
- translates words and phrases into variables, expressions, and equations, with assistance;
- describes patterns in problems, with assistance;
- plots points in a co-ordinate system.

Standards of Performance

Level 2	Level 3	Level 4
The student simplifies algebraic expressions using concrete materials. The student applies patterning and algebra skills to solve problems.	The student simplifies algebraic expressions using a variety of methods and tools. The student develops formulas as generalizations about observed patterns.	The student develops algebraic generalizations about complex patterns and uses patterns to model problems.
In addition, the student:	In addition, the student:	In addition, the student:
– describes, predicts, and verifies a pattern where concrete modelling is practical;	– describes, predicts, and extends a pattern beyond the point where concrete modelling is practical, using variables;	– develops algebraic generalizations about complex patterns;
– simplifies algebraic expressions using concrete materials;	– simplifies algebraic expressions using a variety of methods and tools;	– develops generalizations about the simplification of algebraic expressions (e.g., [ax + bx = (a+b) x]);
– solves problems and linear equations using trial and error, inspection, and concrete materials;	– solves problems and linear equations using formal methods and formulas;	– uses equations and formulas to solve multi-step problems;
– draws graphs using data from real life, with some assistance;	– graphs relations using a variety of methods and technologies, including those from other subject disciplines;	– develops a formula to represent the graph of a relation;
– matches mathematical expressions to the words and phrases they represent;	– translates words and phrases that model real-life situations into mathematical expressions;	– restates complex problems as linear equations and solves related problems;
– identifies and describes patterns in problem-solving activities using a variety of tools (e.g., calculators, computers);	– develops formulas as generalizations about observed patterns;	– uses patterns to model problems;
– graphs relations from a given table of values.	– graphs relations from a table of values developed in an investigation.	– generates a table of values from experiments and formulas and analyses the graph of the relation.

Outcomes from *Policies and Outcomes, Grades 1–9*

By the end of Grade 9, students will:

- gather information from various sources, make decisions based on the information, and communicate the decisions, using appropriate methods;
- use probability and statistics in a variety of investigations.

More specifically, students will:

- establish a plan and procedure for collecting data through surveys or experiments; collect the data; analyse, organize, interpret, and display the information appropriately; and pose and answer questions about the data collected;
- read charts, tables, and graphs and interpret the information;
- carry out a probability experiment or simulation and draw conclusions based on the results;
- construct graphs from given data and pose and answer questions about the graphs;
- calculate the mean, median, and mode from a set of data and explain which measure best represents the population;
- use a variety of sampling techniques to estimate populations and explain why randomness is important to the validity of a sample;
- analyse the use of probability and statistics in the media.

Sample Assessment Activities

1. Toastie T cereal company decides to promote its cereal by including in each box the photo of one of six superheroes. Assuming that there are equal numbers of photos of each of the six superheroes and that when you buy a box of cereal your chances of getting any of the six photos are the same, about how many boxes of cereal would you expect to have to buy to get all six photos?
2. Answer the following questions:
   What is a survey?
   Why would someone want to conduct a survey?
   What kinds of questions are usually asked in a survey?
   What kind of information can be obtained from a survey?
   How can this information be used?

Level 1

Characteristics of Student Performance

The student collects, organizes, and displays data, with assistance. The student carries out a simple probability experiment, records the data, and draws conclusions, with assistance.

Sample Performance Indicators

The student:

- collects, organizes, and displays data when given specific instructions, and answers teacher's questions;
- describes information found in simple charts, tables, and graphs;
- carries out a probability experiment for which the instructions are provided, and records the results;
- constructs graphs, with assistance;
- uses appropriate technology to organize and display data, with assistance;
- calculates mean, median, and mode, with assistance;
- uses random numbers to select a sample, with assistance.

Standards of Performance

Level 2	Level 3	Level 4
The student collects, organizes, and displays data when given instructions and answers specific questions pertaining to displayed data. The student answers questions about results from a probability experiment.	The student applies data-management skills to a variety of situations. The student can explain simple probability concepts and make some generalizations about his or her findings.	The student uses various data-management skills in complex problem-solving situations. The student designs and carries out investigations related to probability and statistics.
In addition, the student:	In addition, the student:	In addition, the student:
– collects, organizes, and displays data when given instructions and analyses and interprets the results, with some teacher direction;	– identifies a question of personal interest or one related to a social issue and collects, organizes, displays, and analyses data related to it;	– poses a question, investigates it, creates a model, and makes informed statements about the conclusion;
– reads scales and labels on graphs and charts; – answers specific questions about displayed data;	– interprets scales and labels correctly on graphs and charts; – uses appropriate units to describe data and patterns extrapolated from them;	– analyses and describes complex relationships displayed in charts, tables, and graphs;
– carries out a probability experiment for which the instructions are provided and records and analyses the results, with some teacher assistance;	– carries out a probability experiment and explains the difference between the experimental and the theoretical outcome;	– formulates a problem and then designs, carries out, and evaluates a probability experiment or simulation;
– constructs graphs from given data and answers questions about the graphs, with assistance;	– selects the appropriate type of graph and units for a particular purpose; – correctly labels and displays data; – answers questions about graphs;	– poses questions about graphs and draws conclusions from the information displayed;
– uses technology to organize and display data;	– uses technology to collect, organize, display, and analyse data;	– compares available technologies for organizing, displaying, and analysing data;
– calculates mean, median, and mode;	– explains which measure of central tendency (mean, median, or mode) best represents the population;	– formulates and states problems that illustrate the different uses of mean, median, and mode;
– uses random numbers to select a sample.	– explains randomness and why it is important in sampling.	– compares various methods of choosing a random sample.

Appendix. Resources Used

The following resources have been used in the preparation of sections of this document:

Ontario Association for Mathematics Education (OAME) and Ontario Mathematics Coordinators Association (OMCA). *Focus on Renewal of Mathematics Education*. Toronto, Ont.: OAME and OMCA, 1993.

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Polya, George. *How to Solve It: A New Aspect of Mathematical Method*. 2nd ed. 1945. Reprint, Princeton, N.J.: Princeton University Press, 1973.

Working Groups of the Commission on Standards for School Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, Va.: National Council of Teachers of Mathematics, 1989.

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